N64-18298
CODE-1
CR-53361

10p.

ELASTIC STABILITY OF CYLINDRICAL SHELLS UNDER AXIAL AND LATERAL LOADS

C. LAKSHMIKANTHAM
GEORGE GERARD

OTS PRICE

t: ELASTIC STABILITY OF CYLINDRICAL SHELLS UNDER AXIAL AND LATERAL LOADS

C. LAKSHMIKANTHAM and
GEORGE GERARD

10 Feb. 10p refs

cite only

NASA CR-53361; TECHNICAL ~~REPORT~~ ~~NO.~~ 235-4
FEBRUARY 10, 1964

OTS: $---

prepared for
(NASA CONTRACT ~~NO.~~ NASw-682)
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON 25, D. C.

0147512

ALLIED RESEARCH ASSOCIATES, INC.
VIRGINIA ROAD CONCORD, MASSACHUSETTS

SYMBOLS

D	=	flexural rigidity
E	=	elastic modulus
k	=	buckling coefficient
L	=	length of the cylinder
N	=	compressive external force
R	=	radius of the cylinder
t	=	thickness of the cylinder
w	=	displacement normal to cylinder surface
Z	=	shell curvature parameter = $L^2/Rt\,(1-\nu^2)^{\frac{1}{2}}$
β	=	ratio of axial to circumferential wavelengths
λ	=	half wavelength of buckle
τ	=	ratio of circumferential to axial compressive forces = N_y/N_x

ELASTIC STABILITY OF CYLINDRICAL SHELLS UNDER AXIAL AND LATERAL LOADS

INTRODUCTION

In the stability analysis of cylinders under external loading, the axial compression and lateral pressure cases are relatively well established: see for example Ref. 1. However, from a design point of view, a biaxial system of forces due to a combination of axial compression and external pressure is often encountered in launch vehicle structures. While many other combined loading cases have appeared in the literature, the case under present consideration has not and therefore this paper is devoted to a general treatment of this problem. It is to be noted that Radhakrishnan (Ref. 2) presented some specific results for this loading combination for elastic and plastic buckling.

Using the Donnell equation for small deformations, the present report considers the effect of various compressive loading combinations on the stability problem of an unstiffened circular cylinder.

GOVERNING EQUATIONS

The Donnell equation for cylinders in terms of w, the normal displacement, in the absence of twisting forces can be written as follows:

$$D \nabla^8 w + \nabla^4[N_x \, \partial^2 w/\partial x^2 + N_y \, \partial^2 w/\partial y^2] + Et/R^2 \partial^4 w/\partial x^4 = 0 \tag{1}$$

where, x and y refer to the axial and circumferential directions respectively.

A suitable form for w, in the above equation would be:

$$w = \sin(x/\lambda_x) \sin(y/\lambda_y) \tag{2}$$

where, λ_x and λ_y are the half wavelengths of buckles along x and y directions respectively.

Now, for the case of axial compression, that is $N_x = N$ and $N_y = 0$, the buckle wavelengths are indeterminate. In other words, one can obtain the buckling stress without making any assumptions on λ_x and λ_y, or one can assume that these are equal to each other. However, an introduction of lateral pressure usually causes a single buckle along the length of the cylinder, while there are many buckles in the circumferential direction. Hence in what follows it is assumed that there is a single buckle along the length of the cylinder. Thus $\lambda_x = L/\pi$ where L is the length of the cylinder. Introducing $\lambda_x/\lambda_y = \beta$, and upon substituting Eq. (2) into Eq. (1) we obtain,

$$N_x L^2/\pi^2 D = (1 + \beta^2)^2/(1 + \beta^2 \tau) + (Et\ L^4/\pi^4 R^2 D)\ 1/(1 + \beta^2 \tau)(1 + \beta^2)^2 \tag{3}$$

or

$$N_x L^2/\pi^2 D = (1+\beta^2\tau) + 12\ Z^2/\pi^4\ 1/(1=\beta^2)^2\ (1+\beta^2\tau)$$

where

$$\tau = N_y/N_x \text{ and } Z = (L^2/Rt)\ (1 - \nu^2)^{\frac{1}{2}}$$

When the expression on the right hand side of Eq. (3) is minimized with respect to β^2, we obtain a quintic in β^2:

$$(1+\beta^2)^4\ (2+\beta^2\tau-\tau) = 12Z^2/\pi^4\ (2+3\beta^2\tau+\tau) \tag{4}$$

While Eq. (4) does not yield a simple algebraic expression for the roots of β^2, one can use it to compute Z for various values of β^2 and τ by re-writing Eq. (4) as:

$$12Z^2/\pi^4 = (1+\beta^2)^4\ (2+\beta^2\tau-\tau)/\ (2+3\beta^2\ \tau+\tau) \tag{5}$$

A substitution of Eq. (5) into Eq. (3) leads to the following expression for the buckling stress coefficient

$$k_x = 4(1+\beta^2)^2/(2+3\beta^2\ \tau+\tau) \tag{6}$$

Using Eqs. (5) and (6) we can obtain a k-Z plot for various values of Z.

RESULTS

Before Eqs. (5) and (6) are solved as such for some special values of β^2 and τ, Eq. (3) can be minimized readily and simple expressions obtained for k_x or k_y in terms of Z. Thus:

(a) $\tau = 0$ (axial pressure) leads to

$$k_x = 0.702\ Z \tag{7}$$

(b) $\tau = 0$ (1) and $\beta^2 >> 1$ leads to

$$k_x = 4\sqrt{6}/3\pi\ (Z^{\frac{1}{2}}/\tau) \text{ or } k_x = 1.038\ (Z^{\frac{1}{2}}/\tau) \tag{8}$$

(c) $\tau = 1$ lead to,

$$k_x = k_y = 1.038\ Z^{\frac{1}{2}} \tag{9}$$

(d) $\tau \to \infty$ $\beta^2 >> 1$ (lateral pressure) leads to

$$k_y = 1.038\ Z^{\frac{1}{2}} \tag{10}$$

Case (a) is valid for moderate length cylinders for which $Z \geq 10$ while (d) is valid for $Z \geq 100$ as in Ref. (1). Hence, we may say that for moderate length cylinders, Z not less than 10^2, the above equations are valid.

Eqs. (5) and (6) have been solved for different values of τ, assigning values of β for each τ. Figure 2 shows a k_x - Z plot for different values on a logarithmic scale. The $\tau = 0$ line represents the axial pressure case. The line $\tau = 1$ is also shown as the $k_y = 1.038Z^{\frac{1}{2}}$ line for the lateral pressure case; this is because as τ becomes ≥ 1, the critical stress is written as k_y instead of k_x. Eq. (8) shows that for all $\tau \geq 1$, and $\beta >> 1$, we obtain $\tau\, k_x = k_y = 1.038\ Z^{\frac{1}{2}}$.

It is interesting to note the influence of τ on the k_x - Z curves. For small τ values and moderate values of Z, they follow the $k_x = 0.702$ Z line and as Z increases in value such that $(\beta^2\ \tau) >> 1$, the slope of the curve decreases until they become parallel to the $Z^{\frac{1}{2}}$ line. This asymptotic behavior is evident from Eq. (3) where if $(\beta^2\ \tau) \gg 1$ for all τ, however small τ be, then a minimization yields $k_x = 1.038\ Z^{\frac{1}{2}}/\tau$. Hence, even for small τ values, as Z becomes sufficiently large the curve becomes parallel to the lateral pressure case and this behavior remains as τ increases.

Figs. (2) and (3) present the data in slightly different forms to bring out the role of τ on the stability problem. Fig. (2) shows a k_y - Z plot, while Fig. (3) plots values of k_x at any τ, normalized with respect to the k_x for the compressive case, against Z.

CONCLUDING REMARKS

From the data presented in the figures, one may conclude that no matter by what loading mechanism the parameter τ is introduced, the characteristic of the compressive buckling problem is affected profoundly. Firstly, the values of buckling stresses are lowered significantly; secondly, there is a Z dependence of the buckling coefficient k_x as contrasted with the compressive case: that is, the slope keeps decreasing as Z increases; finally, for large values of Z the slope of the k_x - Z plot follows the lateral pressure case.

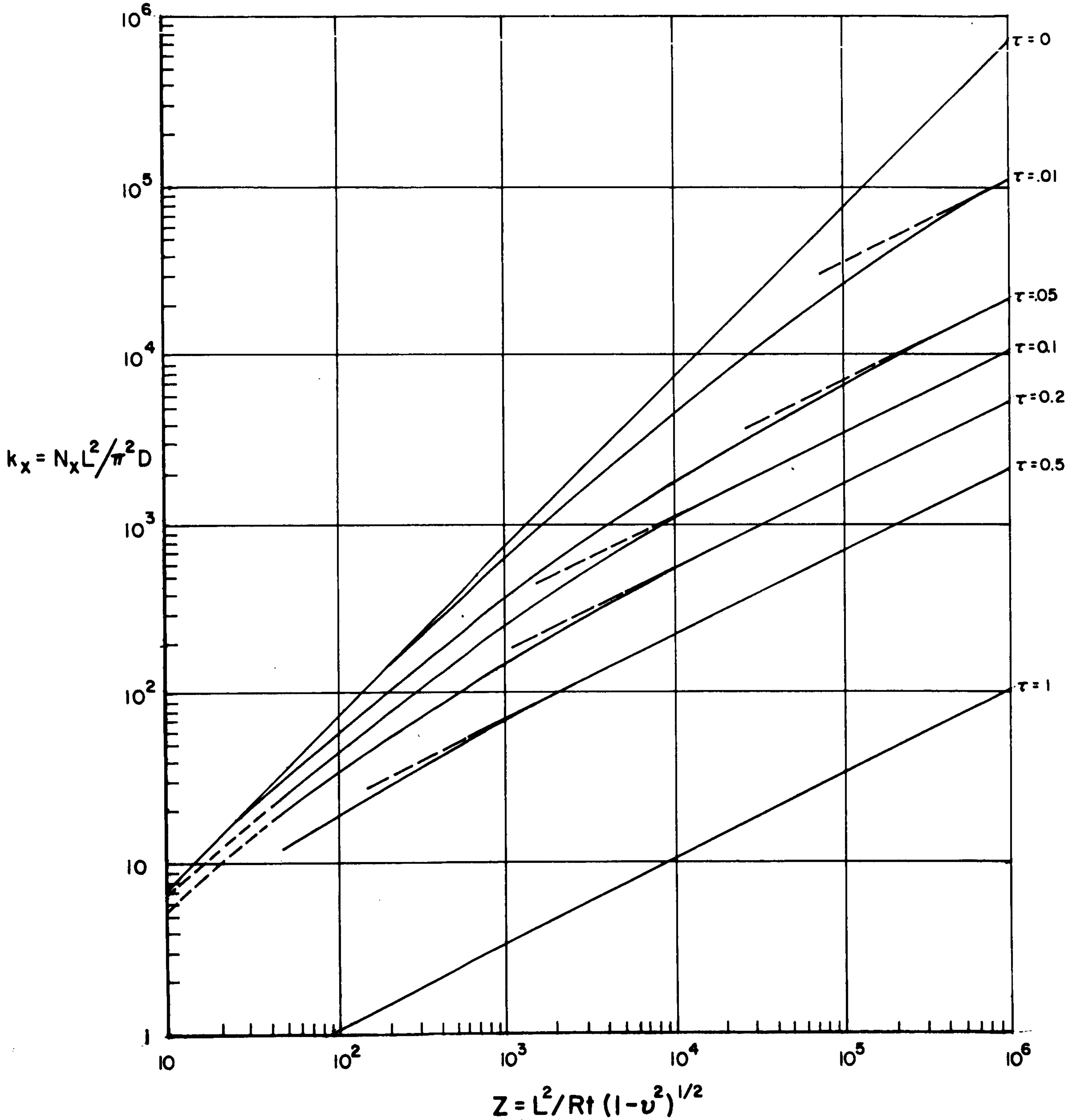
These results have certain interesting implications that pertain to an unstiffened cylinder under axial compressive loads. If it is postulated that a small τ value is generated as a result of initial imperfections or prebuckling deformations, then the results presented in Figs. (2) and (3) indicate an unusual sensitivity to small values of τ. It is interesting to observe that the theoretical results presented in Fig. (2) follow the trend of test data on cylinders segregated according to R/t values. Furthermore, Fig. (3) is suggestive of the usual C vs. R/t plots for test data on unstiffened cylinders.

REFERENCES

1. Gerard, G., Introduction to Structural Stability Theory, McGraw-Hill Book Co., Inc., New York 1962, chs. 7 and 8.

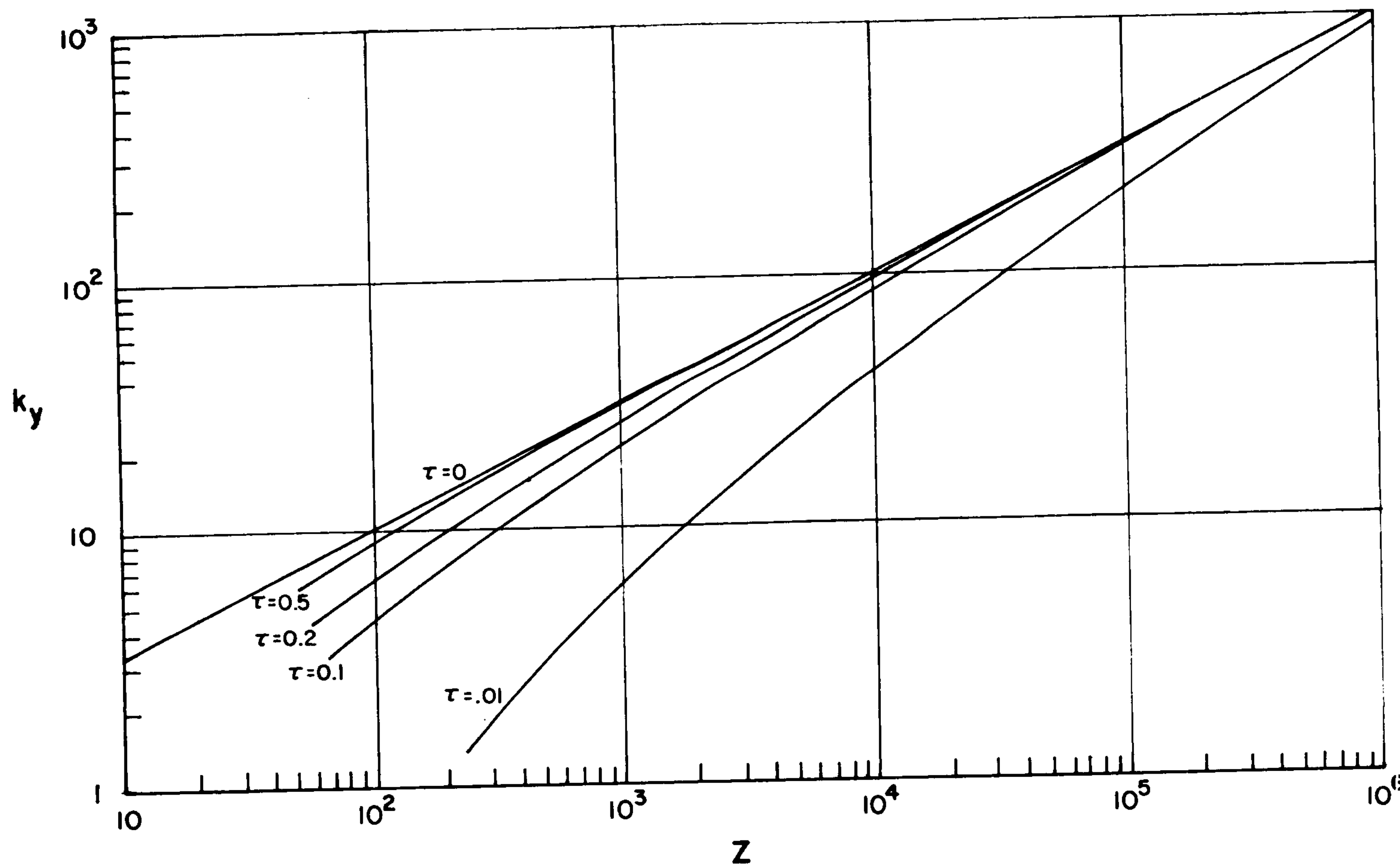
2. Radhakrishnan, S., "Plastic Buckling of Cylindrical Shells," Aircraft Engineering, December 1959, pp. 365-372.

FIG. 1



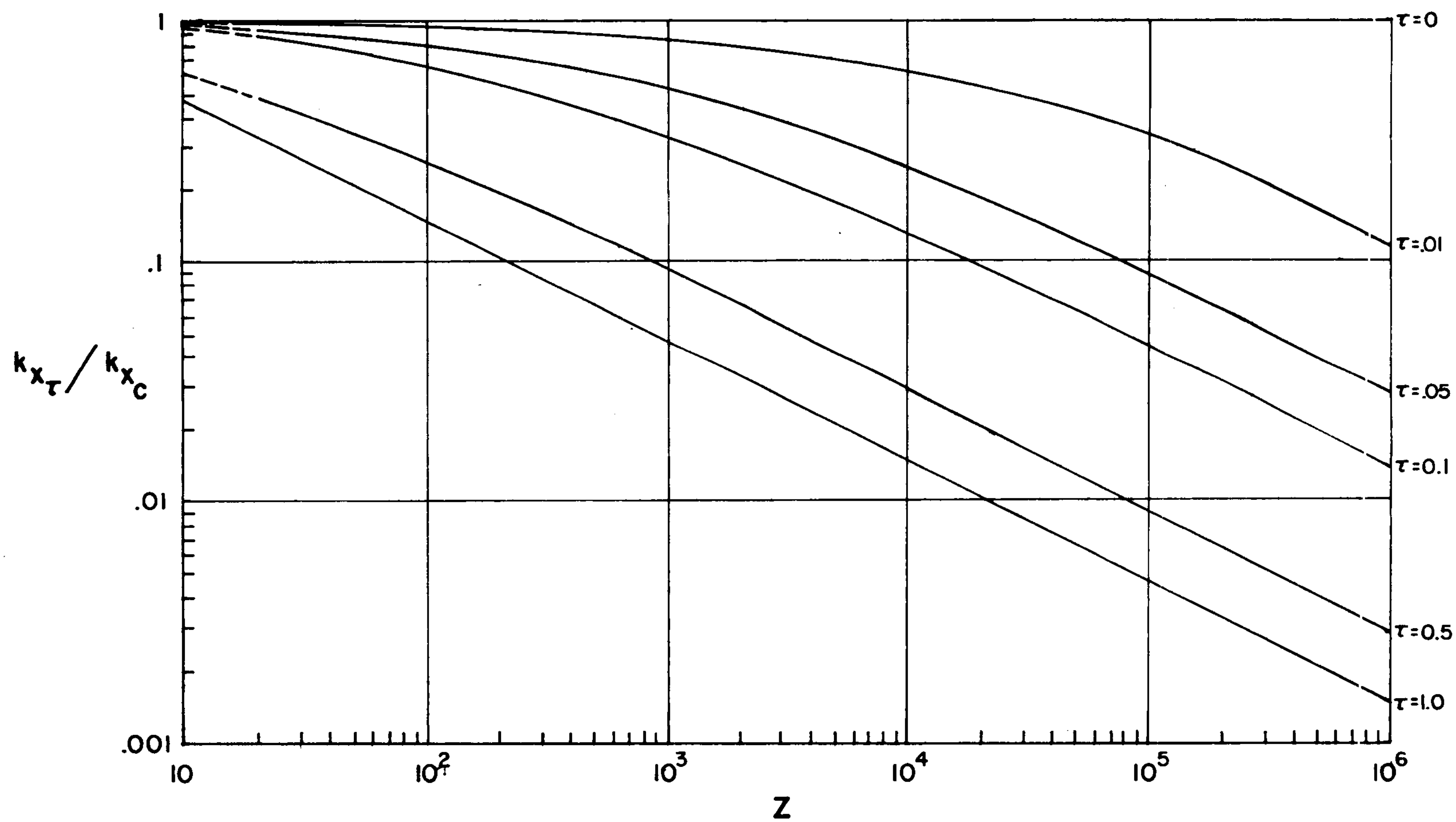
10^6
10^5
10^4
10^3
10^2
10
1
$k_x = N_x L^2/\pi^2 D$
10
10^2
10^3
10^4
10^5
10^6
$Z = L^2/Rt\,(1-\nu^2)^{1/2}$
$\tau = 0$
$\tau = .01$
$\tau = .05$
$\tau = 0.1$
$\tau = 0.2$
$\tau = 0.5$
$\tau = 1$

FIG. 2



k_y
10^3
10^2
10
1
10
10^2
10^3
10^4
10^5
10^6
Z
$\tau=0$
$\tau=0.5$
$\tau=0.2$
$\tau=0.1$
$\tau=.01$

FIG. 3



k_{x_τ}/k_{x_c}
1
.1
.01
.001
10
10^2
10^3
10^4
10^5
10^6
Z
τ=0
τ=.01
τ=.05
τ=0.1
τ=0.5
τ=1.0